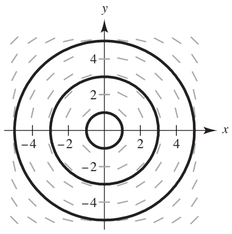
46. $\dfrac{dy}{dx} = -\dfrac{x}{y}$

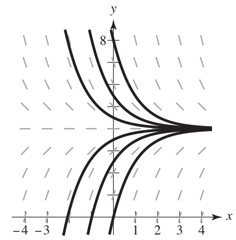


$$y\,dy = -x\,dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C_1$$

$$y^2 + x^2 = C$$

47. $\dfrac{dy}{dx} = 4 - y$



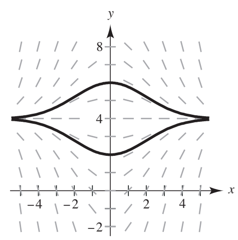
$$\int \frac{dy}{4-y} = \int dx$$

$$\ln|4 - y| = -x + C_1$$

$$4 - y = e^{-x+C_1}$$

$$y = 4 + Ce^{-x}$$

48. $\dfrac{dy}{dx} = 0.25x(4 - y)$



$$\frac{dy}{4-y} = 0.25x\,dx$$

$$\int \frac{dy}{y-4} = \int -0.25x\,dx = -\frac{1}{4}\int x\,dx$$

$$\ln|y - 4| = -\frac{1}{8}x^2 + C_1$$

$$y - 4 = e^{C_1-(1/8)x^2} = Ce^{-(1/8)x^2}$$

$$y = 4 + Ce^{-(1/8)x^2}$$

49. (a) Euler's Method gives $y(1) \approx 0.1602$.

(b) $\dfrac{dy}{dx} = -6xy$

$$\int \frac{dy}{y} = \int -6x$$

$$\ln|y| = -3x^2 + C_1$$

$$y = Ce^{-3x^2}$$

$$y(0) = 5 \Rightarrow C = 5$$

$$y = 5e^{-3x^2}$$

(c) At $x = 1$, $y = 5e^{-3(1)} \approx 0.2489$.

Error: $0.2489 - 0.1602 \approx 0.0887$

50. (a) Euler's Method gives $y(1) \approx 0.2622$.

(b) $\dfrac{dy}{dx} = -6xy^2$

$$\int \frac{dy}{y^2} = \int -6x\,dx$$

$$\frac{-1}{y} = -3x^2 + C_1$$

$$y = \frac{1}{3x^2 + C}$$

$$3 = \frac{1}{C} \Rightarrow C = \frac{1}{3}$$

$$y = \frac{1}{3x^2 + \frac{1}{3}} = \frac{3}{9x^2 + 1}$$

(c) At $x = 1$, $y = \dfrac{3}{9(1) + 1} = \dfrac{3}{10} = 0.3$.

Error: $0.3 - 0.2622 = 0.0378$

51. (a) Euler's Method gives $y(2) \approx 3.0318$.

(b) $\dfrac{dy}{dx} = \dfrac{2x + 12}{3y^2 - 4}$

$$\int (3y^2 - 4)\,dy = \int (2x + 12)\,dx$$

$$y^3 - 4y = x^2 + 12x + C$$

$$y(1) = 2:\ 2^3 - 4(2) = 1 + 12 + C \implies C = -13$$

$$y^3 - 4y = x^2 + 12x - 13$$

(c) For $x = 2$,

$$y^3 - 4y = 2^2 + 12(2) - 13 = 15$$

$$y^3 - 4y - 15 = 0$$

$$(y - 3)(y^2 + 3y + 5) = 0 \implies y = 3.$$

Error: $3.0318 - 3 = 0.0318$

52. (a) Euler's Method gives $y(1.5) \approx 1.7270$.

(b) $\dfrac{dy}{dx} = 2x(1 + y^2)$

$$\int \frac{dy}{1 + y^2} = \int 2x\,dx$$

$$\arctan y = x^2 + C$$

$$\arctan(0) = 1^2 + C \implies C = -1$$

$$\arctan(y) = x^2 - 1$$

$$y = \tan(x^2 - 1)$$

(c) At $x = 1.5$, $y = \tan(1.5^2 - 1) \approx 3.0096$.

53. $\dfrac{dy}{dt} = ky, \quad y = Ce^{kt}$

$y(0) = y_0 = C$ initial amount

$$\frac{y_0}{2} = y_0 e^{k(1599)}$$

$$k = \frac{1}{1599}\ln\left(\frac{1}{2}\right)$$

$$y = Ce^{[\ln(1/2)/1599]t}$$

When $t = 25$, $y = 0.989C$ or 98.9%.

54. $\dfrac{dy}{dt} = ky,\ y = Ce^{kt}$

Initial conditions: $y(0) = 20,\ y(1) = 16$

$$20 = Ce^0 = C$$

$$16 = 20e^k$$

$$k = \ln\frac{4}{5}$$

Particular solution: $y = 20e^{t\ln(4/5)}$

When 75% has been changed:

$$5 = 20e^{t\ln(4/5)}$$

$$\frac{1}{4} = e^{t\ln(4/5)}$$

$$t = \frac{\ln(1/4)}{\ln(4/5)} \approx 6.2 \text{ hr}$$

55. $\dfrac{dy}{dx} = k(y - 4)$

The direction field satisfies $(dy/dx) = 0$ along $y = 4$; but not along $y = 0$. Matches (a).

56. $\dfrac{dy}{dx} = k(x - 4)$

The direction field satisfies $(dy/dx) = 0$ along $x = 4$: Matches (b).

57. $\dfrac{dy}{dx} = ky(y - 4)$

The direction field satisfies $(dy/dx) = 0$ along $y = 0$ and $y = 4$. Matches (c).

58. $\dfrac{dy}{dx} = ky^2$

The direction field satisfies $(dy/dx) = 0$ along $y = 0$, and grows more positive as y increases. Matches (d).

59. $$\frac{dw}{dt} = k(1200 - w)$$

$$\int \frac{dw}{1200 - w} = \int k\,dt$$

$$\ln|1200 - w| = -kt + C_1$$

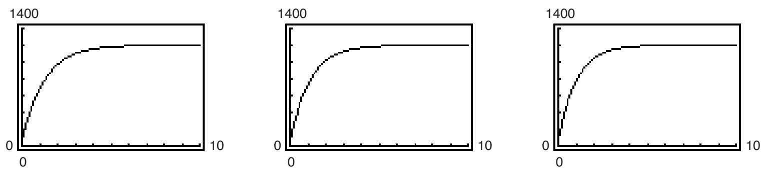
$$1200 - w = e^{-kt + C_1} = Ce^{-kt}$$

$$w = 1200 - Ce^{-kt}$$

$$w(0) = 60 = 1200 - C \implies C = 1200 - 60 = 1140$$

$$w = 1200 - 1140e^{-kt}$$

(a)



(b) $k = 0.8$: $t = 1.31$ years

$k = 0.9$: $t = 1.16$ years

$k = 1.0$: $t = 1.05$ years

(c) Maximum weight: 1200 pounds

$$\lim_{t \to \infty} w = 1200$$

60. From Exercise 101:

$$w = 1200 - Ce^{-kt}, k = 1$$

$$w = 1200 - Ce^{-t}$$

$$w(0) = w_0 = 1200 - C \implies C = 1200 - w_0$$

$$w = 1200 - (1200 - w_0)e^{-t}$$

61. Given family (circles): $x^2 + y^2 = C$

$$2x + 2yy' = 0$$

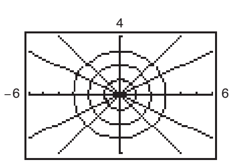
$$y' = -\frac{x}{y}$$

Orthogonal trajectory (lines): $y' = \dfrac{y}{x}$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + \ln K$$

$$y = Kx$$



62. Given family (hyperbolas): $x^2 - 2y^2 = C$

$$2x - 4yy' = 0$$

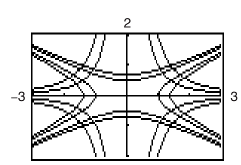
$$y' = \frac{x}{2y}$$

Orthogonal trajectory: $y' = \dfrac{-2y}{x}$

$$\int \frac{dy}{y} = -\int \frac{2}{x}\,dx$$

$$\ln y = -2 \ln x + \ln k$$

$$y = kx^{-2} = \frac{k}{x^2}$$



63. Given family (parabolas): $x^2 = Cy$

$$2x = Cy'$$

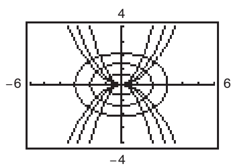
$$y' = \frac{2x}{C} = \frac{2x}{x^2/y} = \frac{2y}{x}$$

Orthogonal trajectory (ellipses): $y' = -\frac{x}{2y}$

$$2\int y\,dy = -\int x\,dx$$

$$y^2 = -\frac{x^2}{2} + K_1$$

$$x^2 + 2y^2 = K$$



64. Given family (parabolas): $y^2 = 2Cx$

$$2yy' = 2C$$

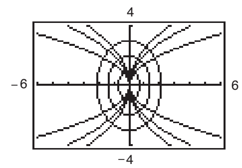
$$y' = \frac{C}{y} = \frac{y^2}{2x}\left(\frac{1}{y}\right) = \frac{y}{2x}$$

Orthogonal trajectory (ellipse): $y' = -\frac{2x}{y}$

$$\int y\,dy = -\int 2x\,dx$$

$$\frac{y^2}{2} = -x^2 + K_1$$

$$2x^2 + y^2 = K$$



65. Given family: $y^2 = Cx^3$

$$2yy' = 3Cx^2$$

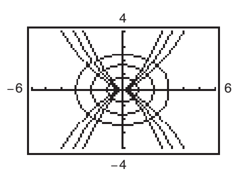
$$y' = \frac{3Cx^2}{2y} = \frac{3x^2}{2y}\left(\frac{y^2}{x^3}\right) = \frac{3y}{2x}$$

Orthogonal trajectory (ellipses): $y' = -\frac{2x}{3y}$

$$3\int y\,dy = -2\int x\,dx$$

$$\frac{3y^2}{2} = -x^2 + K_1$$

$$3y^2 + 2x^2 = K$$



66. Given family (exponential functions): $y = Ce^x$

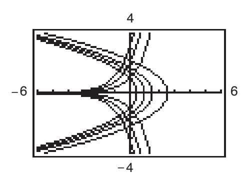
$$y' = Ce^x = y$$

Orthogonal trajectory (parabolas): $y' = -\frac{1}{y}$

$$\int y\,dy = -\int dx$$

$$\frac{y^2}{2} = -x + K_1$$

$$y^2 = -2x + K$$



67. $y = \dfrac{12}{1 + e^{-x}}$

Since $y(0) = 6$, it matches (c) or (d).

Since (d) approaches its horizontal asymptote slower than (c), it matches (d).

68. $y = \dfrac{12}{1 + 3e^{-x}}$

Since $y(0) = \dfrac{12}{4} = 3$, it matches (a).

69. $y = \dfrac{12}{1 + \frac{1}{2}e^{-x}}$

Since $y(0) = \dfrac{12}{\left(\frac{3}{2}\right)} = 8$, it matches (b).

70. $y = \dfrac{12}{1 + e^{-2x}}$

Since $y(0) = 6$, it matches (c) or (d).

Since y approaches $L = 12$ faster for (c), it matches (c).